Imaging and Sleeping Beauty
A case for double-halfers

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Extended abstract

Sleeping Beauty's story is well-known. On sunday evening (t_0), Sleeping Beauty is put to sleep by an experimental philosopher. She is awaken on monday morning and at this moment (t_1), the experimenter doesn't tell her which day it is. Some time later (t_2), she is told that it's monday. At this point, what follows depends on the toss of a fair coin. If the result of the toss is heads, then Sleeping Beauty is put to sleep until the end of the week. If the result is tails, then Sleeping Beauty is awaken on tuesday morning. The crucial fact is that the drug that is given to her is such that she cannot distinguish her awaken on monday from her awakening on tuesday: Sleeping Beauty has a kind of memory erasure.

We are interested in the credence that Sleeping Beauty puts on the proposition that the result of the toss is heads ($HEADS$). More precisely, the two crucial moments are t_1 - when Sleeping Beauty is just awaken on tuesday - and t_2 - when Sleeping Beauty has learned that it's tuesday. I coin the first question Q_1 and the second Q_2. I will adopt the following notation:

- P_1 is Sleeping Beauty's credence at t_1 ie at her awakening on monday morning
- P_2 is Sleeping Beauty's credence at t_2 ie after having learned that it's monday

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What should be the value of $P_1(HEADS)$? There are basically two positions: the *halfers* and the *thirders*. The thirders claim (after A. Elga (Elga 2000)) that $P_1(HEADS) = 1/3$ whereas the halfers claim (after D. Lewis (Lewis 2001)) that $P_1(HEADS) = 1/2$. Now, the answer to Q_1 is intimately linked to the answer to Q_2. As a consequence, the two positions are best described by giving their answer to both questions. By conditionalization, one obtains $P_2(HEADS) = 1/2$ for the thirders and $P_2(HEADS) = 2/3$ for the halfers. We can sum up the positions of Lewis and Elga as follows :

	A. Elga	D. Lewis
Q1	1/3	1/2
Q2	1/2	2/3

Both Elga's and Lewis' basic intuitions are appealing. Elga's intuition is that the coin could be tossed on monday night and that in this case, one should endorse the objective probability of $HEADS$ as her or his credence. Lewis' intuition is that on monday morning, there is no new evidence that is relevant to the credence concerning $HEADS$. Therefore the credence toward $HEADS$ at t_1 should remain the same than at t_0.

The aim of this paper is to propose a case for reconciling these conflicting intuitions. More precisely, I will argue that there is a way to vindicate a *double-halfer* position according to which $P_1(HEADS) = P_2(HEADS) = 1/2$. My case is based on a recent theoretical exploration of probabilistic change rules (Walliser & Zwirn 2002) that shows that whereas bayesian conditionalization may be justified for revising contexts the much less known rule of *imaging* (Lewis 1976) seems to be the appropriate one for updating contexts. Applying the imaging rule instead of bayesian conditionalization in the Sleeping Beauty story results in a double-halfer position.

References

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